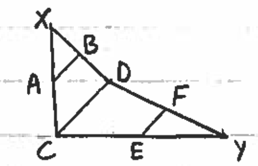
$\boxed{A\#42}$ $\boxed{Pt\ I}$: p. 180-181 WE #1-4, 10-17

$\boxed{Pt\ II}$: p. 182 Self Test I #1-8 and p. 661 #1-2 [2 proofs each]

$\boxed{\text{Key}}$

$\boxed{Pt\ I}$ p. 180-181 WE #1-4, 10-17

For #1-4, A, B, E, and F are the midpoints of $\overline{XC}$, $\overline{XD}$, $\overline{YC}$, and $\overline{YD}$.



For #1-4, $\overline{AB}$ is a midsegment of $\triangle XCD$ and $\overline{EF}$ is a midsegment of $\triangle YCD$. [Def of $\triangle$ midsegment]

1. $CD = 24$ [Given]

   ① $AB = \frac{1}{2}CD$ and ② $EF = \frac{1}{2}CD$ [$\triangle$ midsegment Thm]

   $\boxed{AB = 12}$ and $\boxed{EF = 12}$

2. $AB = K$ [Given]

   ① $AB = \frac{1}{2}CD$ and ② $EF = \frac{1}{2}CD$ [$\triangle$ midsegment Thm]

   $K = \frac{1}{2}CD$ $\quad$ $EF = AB$ [Trans Prop of =]

   $\boxed{CD = 2K}$ and $\boxed{EF = K}$

3. $AB = 5x - 8$ and $EF = 3x$ [Given]

   $AB = \frac{1}{2}CD = EF$ [$\triangle$ Midseg. Thm]

   $5x - 8 = 3x$ [Trans Prop. of =]

   $2x = 8$

   $\boxed{x = 4}$

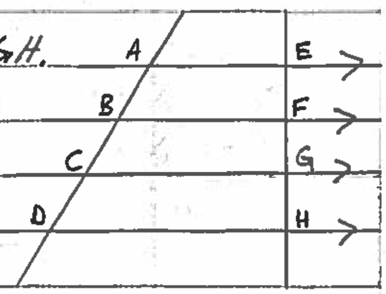
4. $CD = 8x$ and $AB = 3x + 2$ [Given]

   $AB = \frac{1}{2}CD$ [$\triangle$ Midseg. Thm]

   $3x + 2 = \frac{1}{2}(8x)$

   $3x + 2 = 4x \rightarrow \boxed{x = 2}$

For #10-15, $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{DH}$ and $EF = FG = GH$.



For #10-15: ① $\overline{EF} \cong \overline{FG} \cong \overline{GH}$ [Def. of $\cong$ seg.]

② $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ [If 3 or more ∥ lines cut off $\cong$ seg on 1 trans, they cut of $\cong$ seg on all transversals.]

* ③ $AB = BC = CD$ [Def of $\cong$ seg.]

10. $AB = 5$ [Given]

    * $BC = CD = 5$ [Trans. Prop. of =]

    $AD = AB + BC + CD$ [Seg. Add. Post]

    $\boxed{AD = 15}$

11. $AC = 12$ [Given]

    * $AB = BC = 6$ [Trans Prop of = / Seg Add Post]

    * $\boxed{CD = 6}$ [Trans Prop of =]

12. $AB = 5x$ and $BC = 2x + 12$ [Given]

    $5x = 2x + 12$ [Trans Prop. of =]

    $3x = 12 \rightarrow \boxed{x = 4}$

13. $AC = 22 - x$ and $BD = 3x - 22$ [Given]

    * $AC = 2AB$ and $BD = 2AB$ [Seg Add Post / Subst Prop of =]

    $AC = BD$ [Trans Prop of =]

    $22 - x = 3x - 22 \rightarrow 4x = 44 \rightarrow \boxed{x = 11}$

A#42 continued

Key

Pt I p. 180-181 WE #14-17 Pt II p. 182 Self Test I #1-8
p. 661 #1-2

Pt I For #14-15, see the diagram + ①-③ on the previous page.

Continued 14. $AB = 15$, $BC = 2x - y$, and $CD = x + y$ [Given]

*① $2x - y = 15$ [Trans Prop of =]

*② $x + y = 15$ ["]

$3x = 30$ → ② $10 + y = 15$

$\boxed{x = 10}$ $\boxed{y = 5}$

15. $AB = 12$, $BC = 2x + 3y$, and $BD = 8x$ [Given]

① $BD = BC + CD$ [Seg Add Post] and ② $AB = BC = CD = 12$ [Trans Prop of =]

$8x = 12 + 12$

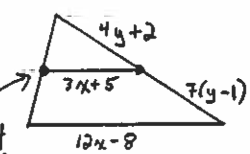
$\boxed{x = 3}$

③ $BC = 2x + 3y = 12$ [Trans Prop of =] → $3y = 6$

$6 + 3y = 12$ $\boxed{y = 2}$

16.

Given: Midpts → + Midsegment (Def of Δ midseg)



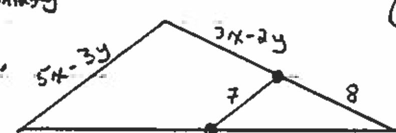
① $4y + 2 = 7(y-1)$ [Midpt Thm] ② $3x + 5 = \frac{1}{2}(12x - 8)$ [Δ Midseg Thm]

$4y + 2 = 7y - 7$ $3x + 5 = 6x - 4$

$9 = 3y$ $9 = 3x$

$\boxed{y = 3}$ $\boxed{x = 3}$

17.



Given: midpts + midsegment [Def of Δ Midseg]

① $3x - 2y = 8$ [Midpt Thm]

② $7 = \frac{1}{2}(5x - 3y)$ [Δ Midseg Thm]

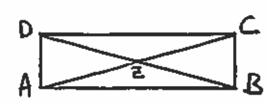
$5x - 3y = 14$ ② $\xrightarrow{\times 2}$ $10x - 6y = 28$

$3x - 2y = 8$ ① $\xrightarrow{\times -3}$ $-9x + 6y = -24$

① $12 - 2y = 8$ ← $\boxed{x = 4}$

$2y = 4$ → $\boxed{y = 2}$

Pt II p. 182 Self Test I #1-8



For # 1-5, ▱ABCD

1. $\overline{AC} \cong \overline{BD}$ – Maybe [Diagonals don't have to be ≅]
2. $\overline{DZ} \cong \overline{BZ}$ – Must be [Diagonals must bisect each other]
3. $\overline{AD} \parallel \overline{BC}$ – Must be [Opposite sides must be ∥]
4. $m\angle DAB = 85°$, $m\angle BCD = 95°$ – Cannot be [Opposite ∠s must be ≅]
5. Prove ▱ABCD
   ① Both Pairs of Opp sides ∥ – $\overline{AD} \parallel \overline{BC}$ and $\overline{DC} \parallel \overline{AB}$
   ② Both Pairs of Opp sides ≅ – $\overline{AD} \cong \overline{BC}$ and $\overline{DC} \cong \overline{AB}$
   ③ One pair of opp sides is both ∥ and ≅ – $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$
   ④ Both pairs of opp. ∠s are ≅ – $\angle ABC \cong \angle ADC$ and $\angle BAD \cong \angle BCD$
   ⑤ All consec. ∠s are Supp – $\angle ADC$ supp to $\angle DAB$, $\angle ADC$ supp to $\angle DCB$
   $\angle DAB$ supp to $\angle ABC$, $\angle ABC$ supp to $\angle DCB$
   ⑥ Diagonals bisect each other – Z is the midpt of $\overline{AC}$ and $\overline{DB}$

A#42 Continued

Pt II p.182 Self Test 2 #6-8 and p.661 #1-2

Key

Pt II 6. a. $3x-7=11$ [If 3 or more // lines cut off ≅ seg on 1 trans, then they cut off ≅ seg on all transversals.]

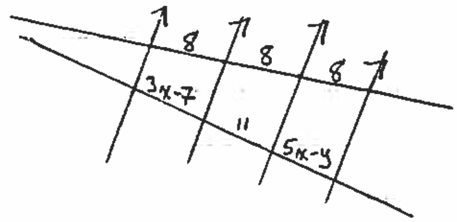
b. ① $3x=18$

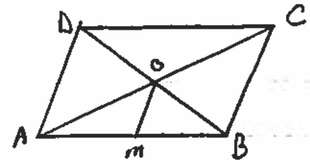
$\boxed{x=6}$

② $5x-y=11$ [Same reason as Part a]

$30-y=11$

$\boxed{y=19}$

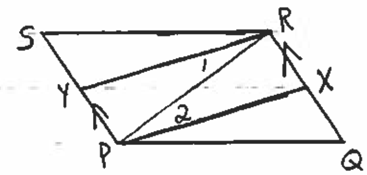




7. Given: ▱ABCD; M is the midpoint of $\overline{AB}$

Prove: $MO = \frac{1}{2}AD$

Statements	Reasons
① ▱ABCD; M is the midpt of $\overline{AB}$	① Given
② $\overline{AC}$ bisects $\overline{BD}$	② Diagonals of a ▱ bisect each other
③ O is the midpt of $\overline{BD}$	③ Def. of seg. bisector
④ $\overline{OM}$ is a midsegment of △ABD	④ Def. of △ midsegment
⑤ $MO = \frac{1}{2}AD$	⑤ △ Midsegment Thm



8. Given: ▱PQRS; $\overline{PX}$ bisects ∠QPR; $\overline{RY}$ bisects ∠SRP

Prove: ▱RYPX

Statements	Reasons
① ▱PQRS; $\overline{PX}$ bisects ∠QPR; $\overline{RY}$ bisects ∠SRP	① Given
② $\overline{SR} \parallel \overline{PQ}$, $\overline{SP} \parallel \overline{RQ}$	② Def of ▱
③ ∠QPR ≅ ∠SRP	③ Alt. Int. ∠s Thm
④ m∠QPR = m∠SRP	④ Def. of ≅ ∠s
⑤ $m\angle 1 = \frac{1}{2} m\angle SRP$, $m\angle 2 = \frac{1}{2} m\angle QPR$	⑤ ∠ bisector Thm (#1)
⑥ $m\angle 2 = \frac{1}{2} m\angle QPR$	⑥ Subst. Prop. of = (4→5)
⑦ ∠1 ≅ ∠2	⑦ Def. of ≅ ∠s
⑧ $\overline{YR} \parallel \overline{XP}$	⑧ Alt. Int. ∠s Converse
⑨ ▱RYPX	⑨ Def. of ▱

$\boxed{A\#42}$ Continued

Key

Pt II p. 661 #1-2 [Demonstrate all 4 methods]
2 proofs each

$\boxed{Pt\ II}$ 1. A(5,7) B(0,3) C(1,-3) D(6,1)

Proof I: Show both pairs of opposite sides are ∥.

m of $\overline{AB} = \frac{\Delta y}{\Delta x} = \frac{7-3}{5-0} = \frac{4}{5}$

m of $\overline{CD} = \frac{\Delta y}{\Delta x} = \frac{1-(-3)}{6-1} = \frac{4}{5}$

m of $\overline{BC} = \frac{\Delta y}{\Delta x} = \frac{3-(-3)}{0-1} = -6$

m of $\overline{AD} = \frac{\Delta y}{\Delta x} = \frac{7-1}{5-6} = -6$

Since the slopes are the same, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$.
∴ ABCD is a ▱.

Proof 2: Show both pairs of opposite sides are ≅.

$AB = \sqrt{(5-0)^2+(7-3)^2} = \sqrt{25+16} = \sqrt{41}$

$CD = \sqrt{(6-1)^2+(1-(-3))^2} = \sqrt{25+16} = \sqrt{41}$

$BC = \sqrt{(1-0)^2+(3-(-3))^2} = \sqrt{1+36} = \sqrt{37}$

$AD = \sqrt{(6-5)^2+(7-1)^2} = \sqrt{1+36} = \sqrt{37}$

Since the lengths are the same, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$.
∴ ABCD is a ▱.

2. A(-2,6) B(-3,2) C(2,-4) D(3,0)

Proof I: Show one pair of opposite sides is both ∥ and ≅.

$AB = \sqrt{(-2-(-3))^2+(6-2)^2} = \sqrt{1+16} = \sqrt{17}$

$CD = \sqrt{(3-2)^2+(0-(-4))^2} = \sqrt{1+16} = \sqrt{17}$

Since the lengths are the same, $\overline{AB} \cong \overline{CD}$.

m of $\overline{AB} = \frac{\Delta y}{\Delta x} = \frac{6-2}{-2-(-3)} = 4$

m of $\overline{CD} = \frac{\Delta y}{\Delta x} = \frac{0-(-4)}{3-2} = 4$

Since the slopes are the same, $\overline{AB} \parallel \overline{CD}$.
∴ ABCD is a ▱.

Proof 2: Show the diagonals bisect each other.

midpoint of $\overline{AC}$ $\left(\frac{-2+2}{2}, \frac{6+(-4)}{2}\right)$
$(0,1)$

midpoint of $\overline{BD}$ $\left(\frac{-3+3}{2}, \frac{2+0}{2}\right)$
$(0,1)$

Since the diagonals have the same midpoint, they bisect each other.
∴ ABCD is a ▱.